On Passive Time-Domain Macromodels of Distributed Transmission Line Networks

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Abstract *– Recently, several time-domain passive macromodeling algorithms were proposed for distributed transmission line networks. Most of them employ some kind of approximation in the frequency-domain to match the response up to a maximum frequency of interest and the behavior after the highest frequency is generally not considered. This can cause significant errors in transient responses (especially in the early-time period). In order to address this difficulty, we will present a new algorithm to reduce these high-frequency errors in time-domain macromodels, while preserving passivity. The proposed algorithm is very useful in eliminating the spurious ripples in the flat delay portion of transient responses of distributed transmission line networks without needing to increase the order of approximation.*

I. Introduction

The ever increasing quest for higher-operating speeds, miniature devices and denser layouts has made the interconnect effects such as delay, crosstalk, ringing and distortion, the dominant factors limiting the overall performance of microelectronic/microwave systems. At higher frequencies, the length of the interconnect becomes a significant fraction of the operating wavelength, and conventional lumped impedance models become inadequate and distributed transmission line models become necessary [1]-[7]. However, simulation of distributed transmission lines in the presence of nonlinear elements suffers from the mixed frequency/time difficulty. There are several techniques available in the literature to address this problem. Broadly speaking, they can be classified into two categories. The first one includes techniques based on the generalized method of characteristics (MC) [2]-[3]. The second category is based on passive macromodeling of transmission lines [4]-[7].

In general, the method of characteristics extracts the line delay and a transfer-function characterizing the frequency response of the line. Subsequently it uses convolution to obtain transient responses. An important advantage of the MC approach is that, since it extracts the line delay explicitly, the corresponding transient responses generally do not exhibit spurious ripples in the early-time region. However, the MC can be CPU expensive in the presence nonlinear elements and lossy coupled lines. In addition, it does not guarantee the passivity of macromodels. Passivity is an important property to satisfy because macromodels that are stable but not passive can produce unstable networks when connected to other passive loads. This can lead to false oscillations during transient simulation.

On the other hand, recently published passive macromodeling schemes [4]-[7] guarantee the passivity of macromodels, and lead to macromodels in terms of ordinary differential equations. Most of these algorithms employ some kind of approximation in the frequency-domain to match the impulse response up to a maximum frequency of interest (f_{max}). However, the behavior after f_{max} is generally not considered, which can lead to significant errors in the impulse transient response (especially in the early-time period) [7]. This can affect the accuracy of the transient response at all other time-points when the macromodel is included during the simulation of a large network. Also, the above problem can be aggravated in the presence of sharp rise times or with smaller capacitive loads. To remove these ripples, the order of the approximation required would be very high, making the macromodel inefficient.

In order to address the above problem, a new algorithm is presented in this paper, which provides a mechanism to control the asymptotic behavior of the high-frequency impulse response while matching the response up to f_{max} accurately. This leads to significant reduction in errors of transient responses. Also, it guarantees the passivity of the macromodel. The proposed algorithm achieves the above objectives with macromodel orders comparable to the ones published in the literature. The macromodel is obtained analytically, in terms of predetermined (stored) constants and the given per-unit length line parameters. Numerical examples are presented to demonstrate the validity, accuracy and efficiency of the proposed model.

II. Proposed Passive Macromodeling Algorithm: Objectives

The objective of the proposed algorithm is to provide a mechanism to control the macromodel impulse response beyond f_{max} so as to minimize early-time ripples while preserving the accuracy and passivity of the macromodel. The early-time impulse response is mainly influenced by the following relationship:

$$h(0^+) = \lim_{s \to \infty} sH_{MN}(s) \tag{1}$$

where 's' is the Laplace operator, $H_{MN}(s)$ represents the frequency-domain rational function, M and N are the numerator and denominator polynomial orders, respectively, and $h(0+)$ represents the early-time response *(around $t=0$)*. Assuming that the k^{th} derivative of the impulse response, $h^{(k)}(0^+) = 0$, then

$$h^{(k+1)}(0^+) = \lim_{s \to \infty} s^{k+2} H_{MN}(s) \tag{2}$$

Observing (1) and (2) one can note that, to obtain flat response around $t=0$, the transfer-admittances represented by $H_{MN}(s)$ must be a *strictly proper rational-function* [8] such that $k = N - M$ is maximum, while preserving the accuracy of the macromodel. Also, it is desired that the order of the denominator *(N)* is kept as minimum as possible for achieving efficient simulation. We will use the above principle to reduce the error in transient responses of distributed transmis-

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sion line macromodels.

III. Review of Distributed Transmission Line Equations

Distributed interconnects are described by a set of partial differential equations known as Telegrapher's equations:

$$\frac{\partial}{\partial x}\boldsymbol{v}(x,t) = -\boldsymbol{R}\boldsymbol{i}(x,t) - \boldsymbol{L}\frac{\partial}{\partial t}\boldsymbol{i}(x,t)$$

$$\frac{\partial}{\partial x}\boldsymbol{i}(x,t) = -\boldsymbol{G}\boldsymbol{v}(x,t) - \boldsymbol{C}\frac{\partial}{\partial t}\boldsymbol{v}(x,t) \quad (3)$$

where $\boldsymbol{R}, \boldsymbol{L}, \boldsymbol{G}, \boldsymbol{C} \in \Re^{\psi \times \psi}$ are the per-unit-length (PUL) parameters of the transmission-line, and are symmetric non-negative definite matrices; $\boldsymbol{V}(x,t), \boldsymbol{I}(x,t) \in \Re^{\psi}$ represent the voltage and current vectors as a function of position x and time t; $\psi + 1$ is the number coupled lines. Equation (3) can be written in the Laplace-domain as

$$\begin{bmatrix} \boldsymbol{V}(d,s) \\ -\boldsymbol{I}(d,s) \end{bmatrix} = e^{\boldsymbol{Z}} \begin{bmatrix} \boldsymbol{V}(0,s) \\ \boldsymbol{I}(0,s) \end{bmatrix}; \ \boldsymbol{Z} = \begin{bmatrix} \boldsymbol{0} & -(\boldsymbol{R}+s\boldsymbol{L})d \\ -(\boldsymbol{G}+s\boldsymbol{C})d & \boldsymbol{0} \end{bmatrix} \quad (4)$$

where $\boldsymbol{V}(s), \boldsymbol{I}(s)$ are the terminal voltage and current vectors of the transmission line and d is the length of the line. Equation (4) does not have a direct representation in the time-domain, which makes it difficult to interface with nonlinear simulators. In [5] a closed-form matrix-rational approximation (MRA) based passive macromodel for distributed transmission lines is suggested, which efficiently addresses the issue of mixed frequency/time simulation. In this paper, we will demonstrate the incorporation of the constraints specified by (1) and (2) using the MRA approach as an example. For this purpose, a brief review of the MRA based macromodeling is given in the next section. However, it is to be noted that, the principles stated in section-II are general in nature and they can also be included in other rational-function based passive macromodeling algorithms available in the literature, with appropriate modifications.

IV. Review of MRA based Passive Macromodels

The exponential matrix $e^{\boldsymbol{Z}}$ in (4) can be expressed with a matrix rational approximation as

$$\boldsymbol{P}_M(\boldsymbol{Z})e^{\boldsymbol{Z}} \approx \boldsymbol{Q}_N(\boldsymbol{Z});$$

$$\boldsymbol{P}_M(\boldsymbol{Z}) = \sum_{i=0}^{M} p_i \boldsymbol{Z}^i; \quad \boldsymbol{Q}_N(\boldsymbol{Z}) = \sum_{i=0}^{N} q_i \boldsymbol{Z}^i \quad (5)$$

where $\boldsymbol{P}_M(\boldsymbol{Z}), \boldsymbol{Q}_N(\boldsymbol{Z})$ are polynomial matrices. The above approximation is formulated analytically in terms of predetermined constants (i.e. q_i and p_i) and PUL parameters. The following theorem was used in [5] to show that for M=N, passive macromodels can be obtained.

Theorem 1: Let the rational-approximation of e^s be

$$e^s \approx \frac{Q_N(s)}{Q_N(-s)} = \left(\sum_{i=0}^{N} q_i s^i\right) \Big/ \left(\sum_{i=0}^{N} q_i (-s)^i\right) \quad (6)$$

where the polynomial $Q_N(s)$ is strictly Hurwitz. *If the above conditions are satisfied, then the rational matrix obtained by replacing the scalar s with the matrix* $\boldsymbol{Z}$ *of* (4) *results in a passive transmission line macromodel* [5]. The form of the resulting matrix-rational approximation can be written as

$$\begin{bmatrix} \boldsymbol{Q}_{N_{11}} & -\boldsymbol{Q}_{N_{12}} \\ -\boldsymbol{Q}_{N_{21}} & \boldsymbol{Q}_{N_{22}} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}(d,s) \\ -\boldsymbol{I}(d,s) \end{bmatrix} \approx \begin{bmatrix} \boldsymbol{Q}_{N_{11}} & \boldsymbol{Q}_{N_{12}} \\ \boldsymbol{Q}_{N_{21}} & \boldsymbol{Q}_{N_{22}} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}(0,s) \\ \boldsymbol{I}(0,s) \end{bmatrix} \quad (7)$$

$$\boldsymbol{Q}_{N_{11}} = \sum_{i=0}^{N} q_i \left[\frac{1}{2}(1+(-1)^i)(\boldsymbol{ab})^{i/2}\right]$$

$$\boldsymbol{Q}_{N_{12}} = \sum_{i=0}^{N} q_i \left[\frac{1}{2}(1-(-1)^i)(\boldsymbol{ab})^{\frac{i-1}{2}}\boldsymbol{a}\right]$$

$$\boldsymbol{Q}_{N_{21}} = \sum_{i=0}^{N} q_i \left[\frac{1}{2}(1-(-1)^i)(\boldsymbol{ba})^{\frac{i-1}{2}}\boldsymbol{b}\right]$$

$$\boldsymbol{Q}_{N_{22}} = \sum_{i=0}^{N} q_i \left[\frac{1}{2}(1+(-1)^i)(\boldsymbol{ba})^{i/2}\right] \quad (8)$$

and $\boldsymbol{a} = \boldsymbol{R} + s\boldsymbol{L}$; $\boldsymbol{b} = \boldsymbol{G} + s\boldsymbol{C}$. Subsequently, the form of the resulting Y-parameters can be written as

$$\begin{bmatrix} \boldsymbol{I}(0,s) \\ \boldsymbol{I}(d,s) \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{11} & \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{21} & \boldsymbol{Y}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}(0,s) \\ \boldsymbol{V}(d,s) \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} \boldsymbol{Y}_{11} & \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{21} & \boldsymbol{Y}_{22} \end{bmatrix} = \boldsymbol{\Psi}_1 + \boldsymbol{\Psi}_2 = \begin{bmatrix} \boldsymbol{H}_a & -\boldsymbol{H}_a \\ -\boldsymbol{H}_a & \boldsymbol{H}_a \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_b \\ \boldsymbol{H}_b & \boldsymbol{H}_b \end{bmatrix}$$

$$\boldsymbol{H}_a = \frac{1}{2}[\boldsymbol{Q}_{N_{12}}]^{-1}\boldsymbol{Q}_{N_{11}} \qquad \boldsymbol{H}_b = \frac{1}{2}[\boldsymbol{Q}_{N_{22}}]^{-1}\boldsymbol{Q}_{N_{21}} \quad (10)$$

The macromodel (9) efficiently captures the response between (0, f_{max}). However, the resulting impulse response of the above macromodel is susceptible to spurious early-time ripples due to the asymptotic behavior of the high-frequency response. This can affect the accuracy of the transient response at all other time-points when the macromodel is included in a large network. In order to address the above problem, a new algorithm is presented in the next section.

V. Development of the Proposed Passive Macromodel

In the new algorithm, we use predetermined coefficients from two different orders of approximation of the scalar exponential matrix in (6), *(N)* and *(N+1)* that satisfy the Theorem-1, to approximate the submatrices of (10) $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$, respectively. Consider a particular order N, if N is even, then the approximation of (6) can be expressed as:

$$e^s \approx \frac{Q_1^{EV} + Q_1^{ODD}}{Q_1^{EV} - Q_1^{ODD}}; \quad Q_1^{EV} = \sum_{i=0}^{N} q_{1,i}\left[\frac{1}{2}(1+(-1)^i)s^i\right], \quad Q_1^{ODD} = \sum_{i=0}^{N} q_{1,i}\left[\frac{1}{2}(1-(-1)^i)s^i\right] \quad (11)$$

Similarly, the other approximation corresponding to $N+1$ order can be denoted as

$$e^s \approx \frac{Q_2^{EV} + Q_2^{ODD}}{Q_2^{EV} - Q_2^{ODD}}; \quad Q_2^{EV} = \sum_{i=0}^{N+1} q_{2,i}\left[\frac{1}{2}(1+(-1)^i)s^i\right] \quad Q_2^{ODD} = \sum_{i=0}^{N+1} q_{2,i}\left[\frac{1}{2}(1-(-1)^i)s^i\right] \tag{12}$$

(please note that if N is odd then the order of (11) is treated as $N+1$ and the order of (12) as N)

The Y-parameters of (10) are obtained by modifying the $Q_{N_{11}}$, $Q_{N_{12}}$, $Q_{N_{21}}$ and $Q_{N_{22}}$ of (8) using (11) and (12) as follows:

$$\begin{aligned}
Q_{N_{11}} &= \sum_{i=0} q_{1,i}\left[\frac{1}{2}(1+(-1)^i)(ab)^{i/2}\right] \\
Q_{N_{12}} &= \sum_{i=0} q_{1,i}\left[\frac{1}{2}(1-(-1)^i)(ab)^{\frac{i-1}{2}}a\right] \\
Q_{N_{21}} &= \sum_{i=0} q_{2,i}\left[\frac{1}{2}(1-(-1)^i)(ba)^{\frac{i-1}{2}}b\right] \\
Q_{N_{22}} &= \sum_{i=0} q_{2,i}\left[\frac{1}{2}(1+(-1)^i)(ba)^{i/2}\right]
\end{aligned} \tag{13}$$

Next, the modified Y-parameters can be re-written as

$$\left.\begin{matrix} Y_{11} \\ Y_{22} \end{matrix}\right\} = \frac{\sum_{i=0}^{N}(\mu_i+\rho_i)(ab)^i}{a\left(\sum_{i=0}^{N-1}\varphi_i(ab)^i\right)}; \quad \left.\begin{matrix} Y_{12} \\ Y_{21} \end{matrix}\right\} = \frac{\sum_{i=0}^{N}(-\mu_i+\rho_i)(ab)^i}{a\left(\sum_{i=0}^{N-1}\varphi_i(ab)^i\right)} \tag{14}$$

The predetermined coefficients μ_i, ρ_i and φ_i in (14) and can be obtained using (11) and (12) as follows.

$$\frac{Q_1^{EV}Q_2^{EV}}{2Q_1^{ODD}Q_2^{EV}} = \frac{\sum_{i=0}^{N}\mu_i s^{2i}}{\sum_{i=0}^{N-1}\varphi_i s^{(2i+1)}}; \quad \frac{Q_1^{ODD}Q_2^{ODD}}{2Q_1^{ODD}Q_2^{EV}} = \frac{\sum_{i=0}^{N}\rho_i s^{2i}}{\sum_{i=0}^{N-1}\varphi_i s^{(2i+1)}} \tag{15}$$

Due to the Hurwitz characteristics of the approximation (6), the coefficients μ_i, ρ_i and φ_i are all positive values. By appropriately choosing the values of ρ_i such that $\mu_i = \rho_i$, (for example $\mu_N = \rho_N$), *the final rational-form of the transfer-admittances* can be obtained with numerator order less than the denominator order $(k > 1)$. The rate of decay can be speeded-up by setting higher values for k (removing more number of zeros from transfer admittances).

Computation of predetermined coefficients

This section describes the formulation of the minimax objective function [5] for predetermining the coefficients of (11) and (12). The method imposes additional constraints to make the transfer-admittances to be strictly proper rational functions with $k > 1$. The goal is to minimize the error function:

$$max_{[f,g]} W(s)\left|e^s - \frac{q_N(s)}{q_N(-s)}\right| \quad is \quad minimum \tag{16}$$

where $W(s)$ is a given weight function, $[f, g]$ is the interval of approximation, $q_N(s)$ is the polynomials of the rational function. The minimax objective function can be obtained by expressing the N^{th} and $(N+1)^{th}$ order approximations given by (11) and (12) in terms of product of second order factors and separating the rational-function into real and imaginary parts, such that the Hurwitz conditions required by Theorem-I are satisfied (details are not given due to the lack of space). Imposing the following additional constraints

$$-\mu_N + \rho_N = 0; \quad -\mu_{N-1} + \rho_{N-1} = 0; \quad \dots \tag{17}$$

removes some of the zeros of Y_{12} and Y_{21} which in turn ensures that the transfer-admittances are strictly proper rational functions with $(k > 1)$. This ensures that the early-time ripples in transient responses are minimized as indicated by (1) and (2).

It should be emphasized that the minimax optimization is performed on the SCALAR function e^s and is independent of the number of coupled lines and the per-unit length parameters. The results obtained are then stored and the macromodel can be obtained analytically in terms of the predetermined coefficients and per-unit-length parameters.

VI. Computational Results

Example 1: Long Lossy Transmission Line

In this example a long lossy distributed transmission line network is analyzed with PRIMA [4] and the proposed algorithm. The near end is connected to a voltage source through a 5Ω resistor and the far end is terminated with a 500Ω resistor. The line is 40cm long with PUL parameters of $R = 1.93\Omega/cm$, $L = 2.97nH/cm$, $G = 79nS/cm$, $C = 1.61pF/cm$. The frequency response at the far-end of the transmission line (Vout) is given in Fig. 1. Both the approaches match the "original response" (obtained by directly solving Telegrapher's equations) up to 2.5GHz, accurately. Note that, the proposed macromodel contains only 31 poles, where as the PRIMA macromodel contains 50 poles. The transient response corresponding to PRIMA macromodel is given in Fig. 2 (the input is a pulse with 0.35ns rise/fall time and 1ns pulse width; the label "IFFT" refers to inverse FFT of "original response" multiplied by the input frequency spectrum). It can be noticed that the flat delay portion of the response from PRIMA suffers from spurious ripples. Fig. 3 gives the transient response of the proposed macromodel and it can be noticed that the spurious ripples in the flat-delay portion are significantly minimized.

Example 2: Lossy Coupled Transmission Line

A network with 2 lossy coupled transmission lines with frequency-dependent parameters is considered (Fig. 4). The time domain responses at the victim node V_4, using the

MRA passive macromodel [5] and the proposed algorithm are given in Fig. 5 and Fig. 6, respectively. As seen, while matching the response accurately, the proposed algorithm minimized the early-time spurious ripples, considerably.

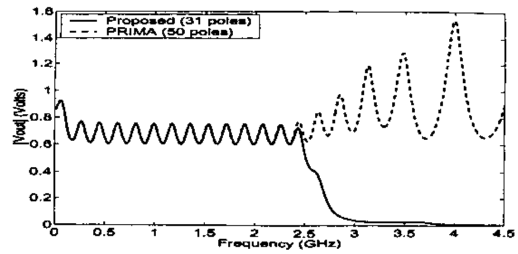

Fig. 1 Frequency Response

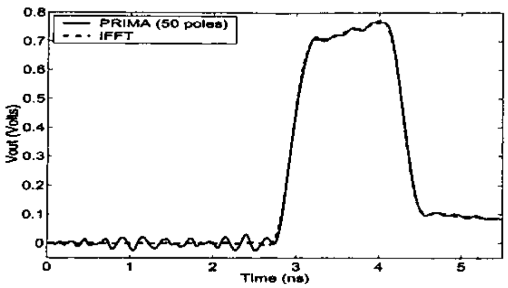

Fig. 2 Time Response (IFFT v/s PRIMA)

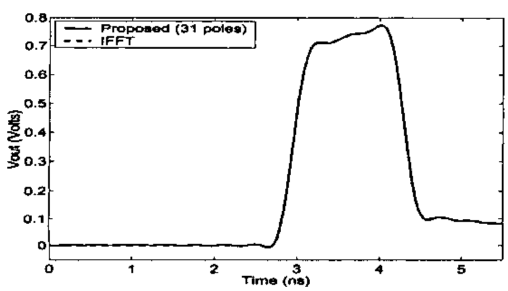

Fig. 3 Time Response (IFFT v/s Proposed Macromodel)

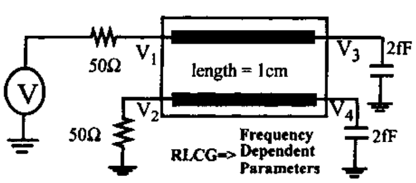

Fig. 4 Lossy Coupled Transmission Line Network

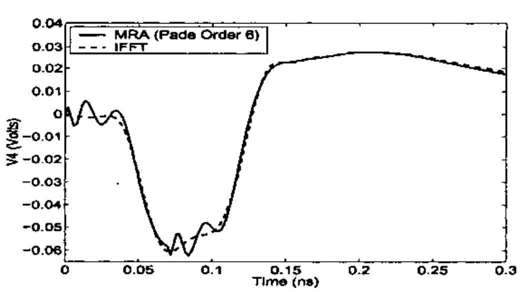

Fig. 5 Time Response (IFFT v/s MRA)

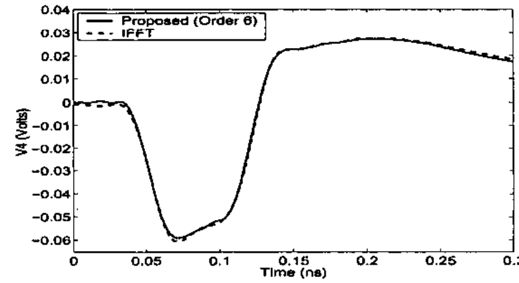

Fig. 6 Time Response (IFFT v/s Proposed)

VII. CONCLUSIONS

In this paper, a new algorithm is presented for accurate passive macromodeling of distributed transmission line networks. The algorithm provides a mechanism to control the asymptotic behavior of the high-frequency impulse response, while matching the response up to f_{max} accurately. This results in significant reduction in early-time spurious ripples in transient responses. It is to be noted that the principles stated in this paper are general in nature and they can be be included in other rational-function based passive macromodeling algorithms available in the literature, with appropriate modifications.

ACKNOWLEDGEMENT

The authors would like to acknowledge I. M. Elfadel, A. E. Ruehli (both with IBM T. J. Watson Research Center, Yorktown Heights, NY) and H-M. Haung (IBM Microelectronics Division, Hopewell Junction, NY) for several useful technical discussions and for providing the test examples.

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